# EEG COHERENCE AND PHASE DELAYS: COMPARISONS BETWEEN SINGLE REFERENCE, AVERAGE REFERENCE AND CURRENT SOURCE DENSITY

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**Abstract**

Mathematical evaluations of coherence and phase differences were conducted using different signal-to-noise ratios based on mixtures of sine waves and random noise. Comparisons were conducted between linked ears reference, single or a common reference, average reference and the Laplacian reference. The results of the analyses demonstrate that linked ears and common references provide valid and systematic relations between the magnitude of coherence and signal-to-noise ratio. In contrast, an average reference and Laplacian reference fail to systematically vary as a function of the signal-to-noise ratio. The reasons for the failure of the average reference and Laplacian is because both methods involve mixing signals from each channel into all other channels whereas a common reference and linked ears do not add signals to other channels.

## 1.0 – Introduction

EEG coherence is a measure of the degree of association or coupling of frequency spectra between two different time series while EEG phase delays are a measure of the temporal "lead" or "lag" of spectra. Mathematically coherence is defined as the normalized cross-power spectrum and phase delay as the "phase angle" and it is computed between two simultaneously recorded EEG signals from different scalp locations per frequency band. Coherence is often interpreted as a measure of "coupling" and as a measure of the functional association between two brain regions (1 Sklar et al, 1972; 2 Shaw et al, 1976; 3 1978; 4 Nunez, 1981; Nunez et al, 1997 5). Coherence is a sensitive measure that can reveal subtle aspects of the network dynamics of the brain which complement the data obtained by power spectral analyses. For example, coherence has been applied to studies of cognition (6 Beaumont et al, 1978; 7 Gevins et al, 1987; 8,9 Thatcher et al, 1983; 1987; 10 Petsche et al, 1993; 11 Rappelsberger and Petsche, 1988), brain maturation (Thatcher et al, 1987 13, Gasser et al, 1988 14; van Baal et al, 1999 15) spatial tasks (Rappelsberger and Petsche 11), agenesis of the corpus callosum (Nielsen et al, 1993 16; Koeda et al, 1995 17; and various clinical populations (Hughes and John, 1999; John et al, 1988 20, Thatcher et al, 1998 ).

EEG phase delays are often used to compute "directed coherence" which is a measure of the directional flow of information between two EEG electrode sites (21, 22). Similar to coherence, EEG phase delays have been used in the analysis of cognition (21, 22) and a variety of clinical conditions (Thatcher et al, 1991 ).

EEG coherence and EEG phase delays are statistical estimates and are dependent on the number of degrees of freedom used to smooth or average spectra as well as the "reference" used to derive the raw digital data.   Thus, while there is only one general mathematical equation for the computation of coherence, nonetheless, differences in the accuracy and sensitivity of the computation of coherence and phase delays depends on the amount of averaging across frequency bands or across records, e.g., "replications" (   ).     Fein et al (1988 26) showed that interhemispheric coherence was inflated when a common reference with a strong signal was used such as Cz.   The authors also computed coherence using reconstructed "reference-free" signals using the source derivation method of Hjorth (1975 27) with ambiguous results.   As pointed out by Rappelsberger (1989 28) source derivation is not "reference free" because it measures the difference between the potential at one electrode with respect to the weighted average potential surrounding that electrode.   Another commonly used EEG reference is the "average reference" in which the average of the potentials from all electrodes is subtracted for the electrical potentials at each electrode.

Rappelsberger (1989 28) used EEG simulations to evaluate EEG coherence recorded with a single reference, the average reference and source derivation and concluded that EEG coherence is invalid using either the average reference or source derivation, e.g., "Tremendous distortions of the theoretical assumptions of common average reference recording and by source derivation are found in the coherence maps" pg. 66.  This same conclusion was arrived at by A. Korzeniewska, M. Mańczak, M. Kamiński, K. Blinowska, S. Kasicki *Determination of information flow direction between brain structures by a modified Directed Transfer Function method (dDTF)* Journal of Neuroscience Methods **125**, 195-207, 2003

The purpose of this paper is to explore and extend the findings of Rappelsberger, 1989 (28) by comparing EEG coherence as well as EEG phase delays using a single reference, an average reference and a source derivation.  Simulations of EEG will involve the use of a systematic mixture of sine waves mixed with noise and recorded from the 19 channels of the international 10/20 system.

**2.0 – Methods**
**2.1 – EEG Simulations**

The EEG was simulated by mixing a known sine wave (1 to 10 uV) at a sample rate of 128 Hz at a given scalp location with varying amplitudes of random gaussian noise (0 to 100 uV) and a constant phase delay of 30 degrees..   The sine waves are defined as:

Eq.1:   x = standard sine wave equation and amplitude, frequency and phase parameters

Where xx is the parameter that controls amplitude, xx controls frequency and xx controls phase shifts.

Coherence was evaluated by comparing a single channel with a pure10 uV sine wave (Fp1) with respect to the remaining 18 channels of the 10/20 electrode system in which varying amounts and this same sine wave was mixed with varying amplitudes of gaussian random noise.   Signal-to-Noise (S/N) was defined as the amplitude of the pure sine wave divided by the amplitude of noise.

Figure 1 is an example of the simulated EEG sine waves in which Fp1 = 10 uV and Fp2 = 10 uV with a 30 degree phase shift and no noise.



Figure 2 is an example of the simulated EEG sine waves in which Fp1 = 10 uV and Fp2 = 10 uV with a 30 degree phase shift and 10 uV of white noise added to Fp2.

## 10 uV Signal + 30 degree Phase Shift + 10 uV Noise
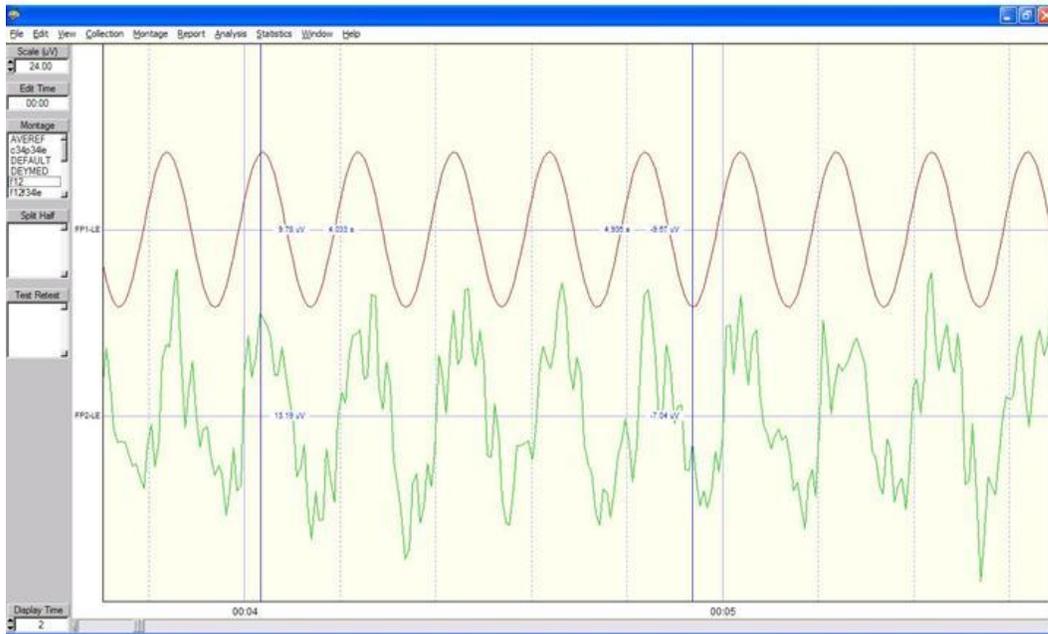


Figure 3 is an example of the simulated EEG in which Fp1 = 10 uV, Noise = 0; Fp2 = 10 uV, Noise = 0, F3 = 10 uV, Noise = 1 uV; F4 = 10 uV, Noise = 2 uV; C3 = 10 uV, Noise = 3 uV; C4 = 10 uV, Noise = 4 uV . . . . . to Pz = 10 uV, Noise = 18 uV.   The range of S/N was from 10 to .01 involving

200 1 uV noise increments.

**10 uV Signal (p-p) + 0 to 18 uV Noise (p-p)**

| Single Reference | Average Reference | Current Source Derivation |
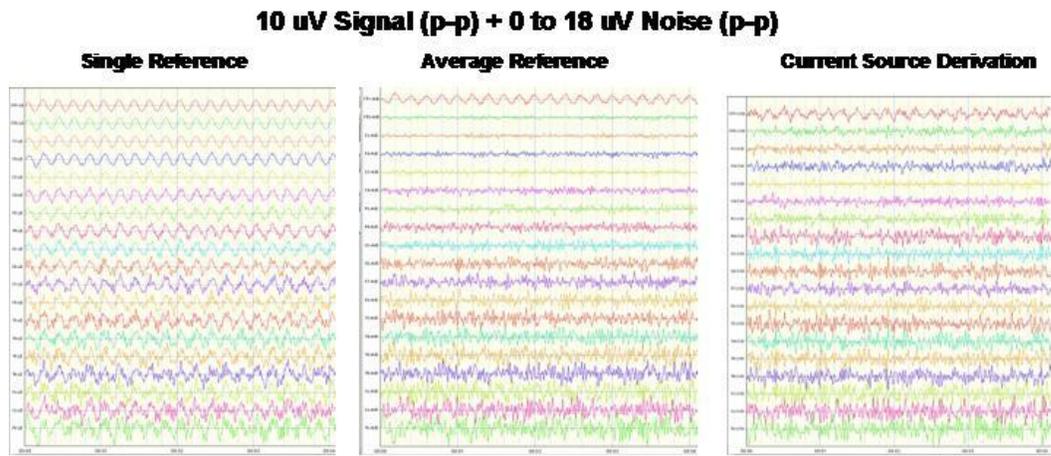


Figure 4 is the FFT of the signals in figure 3.

## FFT of 10 uV Signal (p-p) + 0 to 18 uV Noise (p-p)

| Single Reference | Average Reference | Current Source Derivation |
|---|---|---|
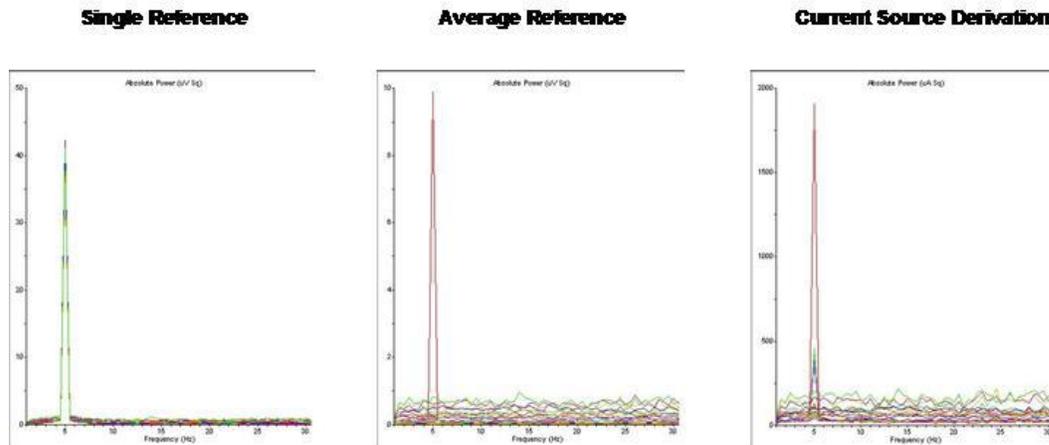


## 2.2 – EEG Reference Computations

The single reference was defined as $A_i - X_i$ , where $A_i$ = an ideal reference and $X_i$ is a single EEG channel for each time point i.   Average reference is defined as $A_i - X_i$, where $A_i = \sum_N X_i$ .  Where N = 19 electrode channels.  In other words, the average electrical potential at each time point is subtracted from each electrode at each time point.   The surface Laplacian or current source density (CSD) was computed using the spherical harmonic Fourier expansion of the EEG scalp potentials to estimate the CSD directed at right angles to the surface of the scalp in the vicinity of each scalp location (Pascual-Marqui, Gonzalez-Andino, Valdes-Sosa, & Biscay-Lirio, 1988 29 ). The CSD is the second spatial derivative or Laplacian of the scalp electrical potentials which is independent of the linked ear reference itself.  The Laplacian is free of a single reference but it is not reference free because it is dependent upon the electrical potential gradients surrounding each electrode.

## 2.3 – Computation of Coherence

The sample length of the simulated EEG samples = 1 minute, sample rate = 128 and the FFT was computed using 2 second overlapping windows with a 75% overlap as described by Kaiser and Sterman

(1999).    This provided a total of 117 windows that were subjected to cross-spectral analyses.
Coherence is mathematically defined two channels X and Y as:

Eq. 2 - Coherence (f) = $\dfrac{\left|Cross - Spectrum(f)XY\right|^2}{(Autospectrum(f)(X))(Autospectrum(f)(Y))}$

However, this standard mathematical definition of coherence hides some of the essential statistical
nature and structure of coherence.   The algebraic notation of equation 3 helps illustrate the fundamental
statistics of coherence:

Eq. 3 -

Coherence (f) = $\dfrac{(\sum_{N}(a(x)u(y)+b(x)v(y)))^2 + (\sum_{N}(a(x)v(y)-b(x)u(y)))^2}{\sum_{N}(a(x)+b(x)^2)\sum_{N}u(y)^2+v(y)^2)}$

and:

$a(x)$ = cosine coefficient for the frequency (f) for channel x
$b(x)$ = sine coefficient for the frequency (f) for channel x
$u(y)$ = cosine coefficient for the frequency (f) for channel y
$v(y)$ = sine coefficient for the frequency (f) for channel y

Where N and the summation sine represents averaging over frequencies in the raw spectrogram or
averaging replications of a given frequency or both.   The numerator and denominator of coherence
always refers to smoothed or averaged values, and, when there are N replications or frequencies then
each coherence value has 2N degrees of freedom.  Note that if spectrum estimates were used which were
not smoothed or averaged over frequencies nor over replications, then coherence = 1 (Bendat and
Piersol, 1980; Benignus, 1968; Otnes and Enochson, 1972).  In order to compute coherence, averaged
cospectrum and quaspectrum smoothed values with degrees of freedom $> 2$ and error bias = 1/N must
be used (xxx).  In the EEG simulations in this study spectra were averaged across 0.5 Hz frequency
bands from 4 to 7 Hz, N = 7 and across replications N = 117 and the degree of freedom = 124 x 2 = 248.
The error bias = 1/248 = .00403

**2.4 – Computation of Phase Delays**

Coherence and phase angle are linked by the fact that the average temporal consistency of the phase angle between two EEG time series is directly proportional to coherence. For example, when coherence is computed with a reasonable number of degrees of freedom and approaches unity, then the phase angle between the two time-series becomes meaningful because the confidence interval of phase is a function of the magnitude of the coherence and the degrees of freedom. If the phase angle is random between two time series then coherence = 0. Another way to view the relationship between phase consistency and coherence is to consider that if Coherence = 1, then once the phase angle relation is known the variance in one channel can be completely accounted for by the other. The phase relation is also critical in understanding which time-series lags or leads the other or, in other words the direction and magnitude of the delay.

The phase angle is defined as:

Eq. 4: Phase angle (f) = Arctan $\dfrac{(Smoothed\,quadspectrum(f))}{(Smoothed\,\cos\,pectrum(f))}$

Where the quadspectrum and cospectrum are defined using the same algebraic notation as in equation xx:

Eq. 5 : $Cospectrum(f) = \sum_{N}(a(x)u(y) + b(x)v(y))$

Eq. 6: $Quadspectrum(f) = \sum_{N}(a(x)v(y) - b(x)u(y))$

As mentioned previously, the confidence internal for the estimation of phase delay or phase angle is directly related to the magnitude of coherence. This means that when coherence is too low, e.g., $< 0.2$, then the estimate of the phase angle may not be reliable.

## 3.0 – Results

### 3.1 – EEG Coherence and Phase using a single reference

Figure 2 are the results of the computation of EEG coherence and EEG phase delays using the single reference EEG simulation.   The y-axis in figure 2 (top) is coherence and the x-axis is the signal-to-noise ratio (S/N).   The y-axis in figure 2 (bottom) is phase delay (degrees) and the x-axis is the same signal-to-noise ratio (S/N) as in figure xA.     It can be seen in Figure 2 that coherence decreases as a linear
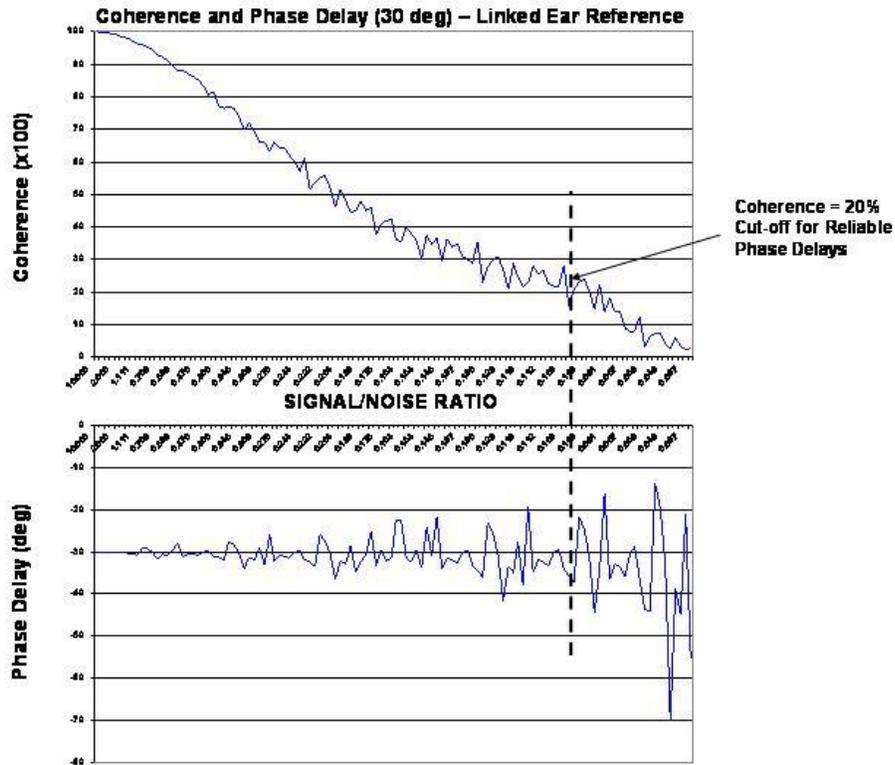


function of signal-to-noise ratio.   It can also be seen in Figure 2 that an approximate average the 30 degree phase delay is present as signal-to-noise increases, however, the variance of the phase delay increases as a function of S/N, especially at coherence values < 0.2.

Figure 3 shows the relationship between the standard deviation of coherence and phase delays as a function of signal-to-noise.   These results show that EEG coherence standard deviations are relatively stable as S/N increases, however, phase delay standard deviations dramatically increase when coherence < 0.2.

COHERENCE & PHASE STDEVs
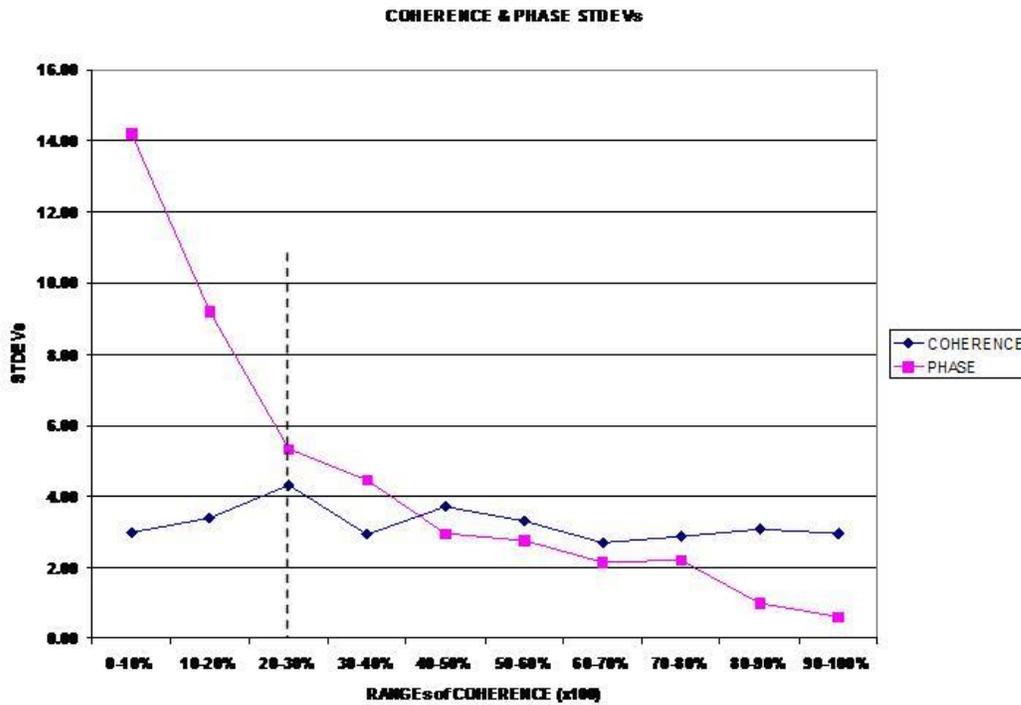


## 3.2 - EEG Coherence and Phase using the average reference

Figure 4 are the results of the computation of EEG coherence and EEG phase delays using the average reference EEG simulation.   The y-axis in figure 4 (top) is coherence and the x-axis is the signal-to-noise ratio (S/N).   The y-axis in figure 4 (bottom) is phase delay (degrees) and the x-axis is the same signal-to-noise ratio (S/N) as in figure 2.     It can be seen in Figure 4 that coherence is extremely variable and does not decrease as a linear function of signal-to-noise ratio.   It can also be seen in Figure 4 that EEG phase delays never approximate 30 degrees and are extremely variable at all levels of noise.

**Coherence – Average Reference**



**Phase 30 deg – Average Reference**
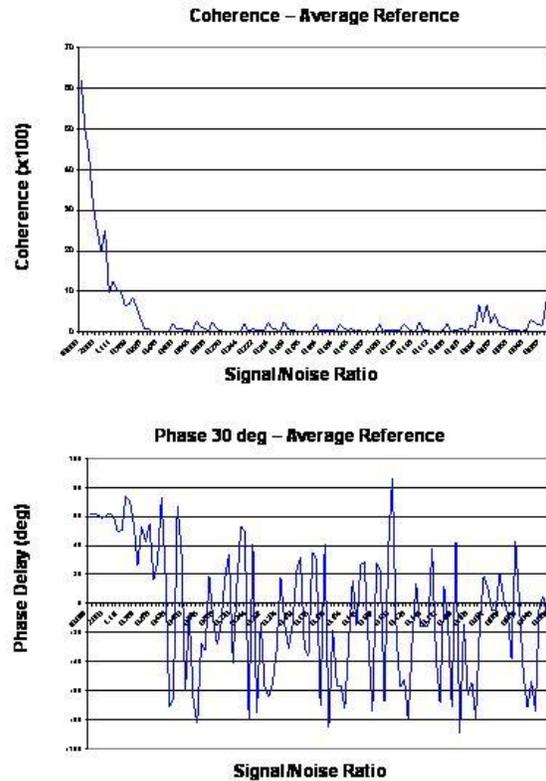


## 3.3 -  EEG Coherence and Phase using the source derivation

Figure 5 are the results of the computation of EEG coherence and EEG phase delays using the average reference EEG simulation.   The y-axis in figure 5 (top) is coherence and the x-axis is the signal-to-noise ratio (S/N).   The y-axis in figure 5 (bottom) is phase delay (degrees) and the x-axis is the same signal-to-noise ratio (S/N) as in figure 2.     It can be seen in Figure 5 that coherence is extremely variable and does not decrease as a linear function of signal-to-noise ratio.   It can also be seen in Figure 5 that EEG phase delays never approximate 30 degrees and are extremely variable at all levels of noise.

**Coherence – Current Source Density**



**Phase 30 deg – Current Source Density**
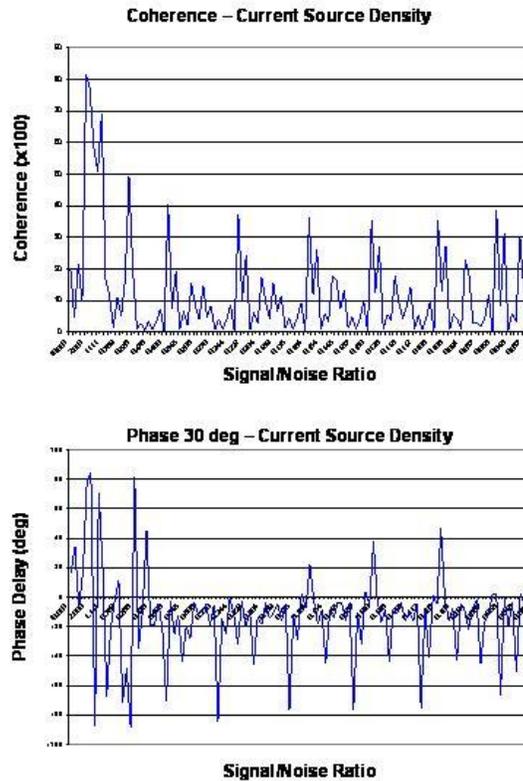


## 4.0 – Discussion

The choice of reference in the computation of EEG coherence is important in quantitative EEG. Studies by Beaumont and Rugg (1979) and French and Beaumont (1984) emphasize the fact that any signal contained in a single reference will be shared by all of the scalp EEG electrodes, depending on the location of the single reference also called a "common reference". Coherence, however, is amplitude independent thus the absolute magnitude of the EEG signal in a single or common reference is irrelevant, except where the magnitude of the signal is very large, such as a Cz reference (Fein et al, 19xx). The results of this study are consistent with those by Rappelsberger, 1989 who emphasized the value and validity of using a single reference and linked ears in estimating the magnitude of shared or coupled activity between two scalp electrodes. The use of re-montage methods such as the average reference and Laplacian source derivation are useful in helping to determine the location of the sources of EEG of different amplitudes at different locations. However, the results of this study which again confirm the findings of Rappelsberger, 1989 show that coherence is invalid when using either an average reference or a source derivation which is also arrived at in a later study by A. Korzeniewska, M.

Mańczak, M. Kamiński, K. Blinowska, S. Kasicki *Determination of information flow direction between brain structures by a modified Directed Transfer Function method (dDTF)* Journal of Neuroscience Methods **125**, 195-207, 2003

It is easy to understand why coherence is invalid when using an average reference since the summation of signals from all channels is "subtracted" or 'added' to the electrical potentials recorded at each electrode. In the present study, a comparison of Figure 2 and figure 4 shows the results of the average reference where noise and signal from each channel is incorporated into all of the channels by being "subtracted" from the electrical potential recorded from each channel. Thus, signals and noise are mixed and added to the recordings from each channel making coherence and phase delays invalid. A similar situation prevails with source derivation since spatially weighted signals and noise from other channels are averaged and subtracted from the electrical potential recorded from each electrode site. Coherence when using the average reference or source derivation is especially sensitive to the presence of artifact or noise since the artifact will be mixed with and added to all channels.

The results of this study also show that measures of phase delay are invalid when using average reference or source derivations. The same arguments for the average reference can be applied to source derivation as they pertain to EEG phase delays. An important finding in the present study concerns the relationship between coherence and phase delays using a single reference or common reference in which the variance of the phase angle dramatically increases at low levels of coherence. We have designated a threshold value of coherence $< 0.2$ below which EEG phase is highly variable and misleading. Although mean phase delay remains at 30 degrees even at high levels of noise (see figures 2 & 3), nonetheless, the extreme variability invalidates EEG phase delays at low levels of coherence (e.g., $<$ 0.2). A similar finding was reported by xxx and xxx. XXX suggested a coherence threshold $< 0.4$, however, this value was a result of using lower degrees of freedom than in the present study for the computation of coherence. It is important to use as high a number of degrees of freedom as possible since, in this study, coherence values were valid even when the signal-to-noise $= 0.01$. It is also important to recognize that estimates of EEG phase delays are extremely variable for low values of coherence, e.g., $< 0.2$ when using a single or common reference.

## 5.0 – References